AP STAT: Section 1.3

Other Distributions (using mean and std. deviation)

Histogram/stemplot/etc.

- samples
- actual observations
- different graphs
- calculate:

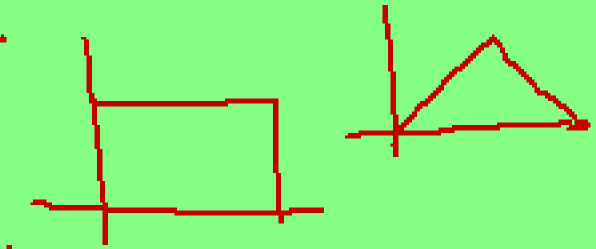
$\bar{x}$, s } sample

$N(\mu, \sigma)$

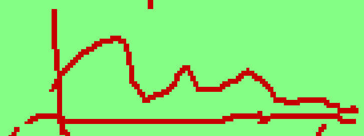
$N(75, 7)$

Density Curve

- different forms



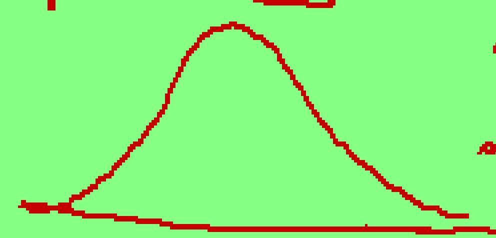
- population



- show overall/general pattern
- smooth curve
- relative freq. (%)
- area under curve = 1 = 100%

Specific Density Curve:

* NORMAL CURVE (or DISTRIBUTION)



- symmetric
- bell-shaped mounded
- unimodal

Ex: ht, weights, test scores

Empirical Rule

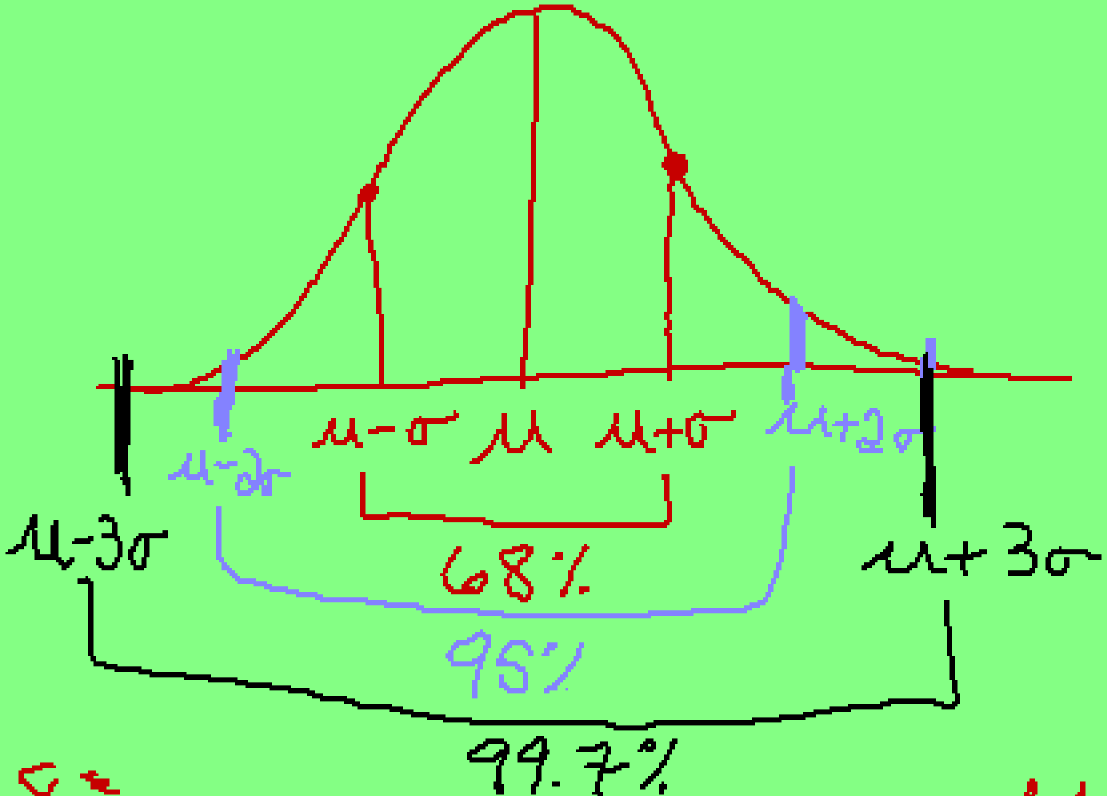
In a normal distribution with N(μ, σ)….

$N(15, 3)$

$(12, 18)$

$(9, 21)$

$(15, 18)$



- 68 % of the observations fall within $\mu \pm \sigma$
- 95 % of the observations fall within $\mu \pm 2\sigma$
- 99.7 % of the observations fall within $\mu \pm 3\sigma$

Example:

The avg. time that it takes a crew from Sweeper Sam to clean a room is normally distributed with a mean of 2.1 hrs and a standard deviation of 0.3 hrs.

$N(2.1, 0.3)$

a) The total clean up time will fall within what interval 95% of the time?

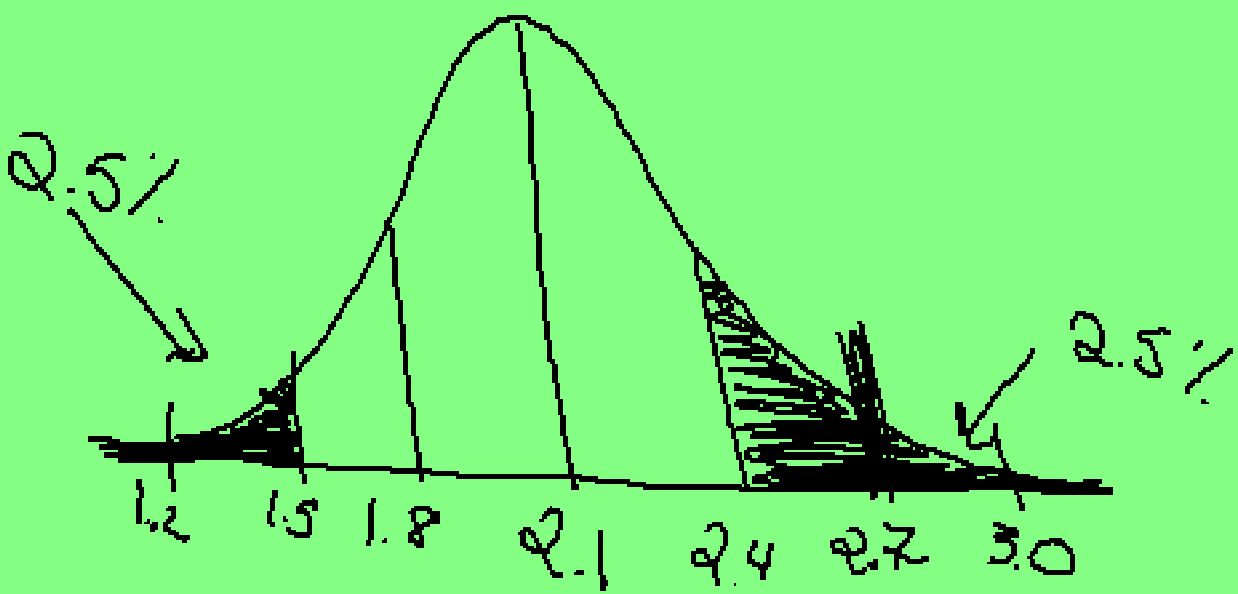
(1.5, 2.7) hrs. $\pm 2\sigma$

b) What proportion of the time will it take the crew [illegible] hours or more?

change to 2.4 hrs: 16%

c) What percent of the time will it take the crew 1.5 hrs or less?

2.5%



Standardizing Observations

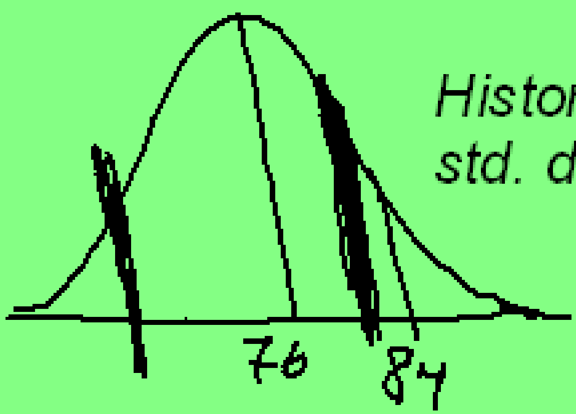
1) You are in a history class and a math class. You take your chapter 1 test in both classes on the same day, and get them both back a few days later. Your grades are as follows. Which class did you do better in?

History: 81%   *Math: 75%*

2) Same scenario. However, each teacher tells you a bit about how your class did. Which class did you do better in?

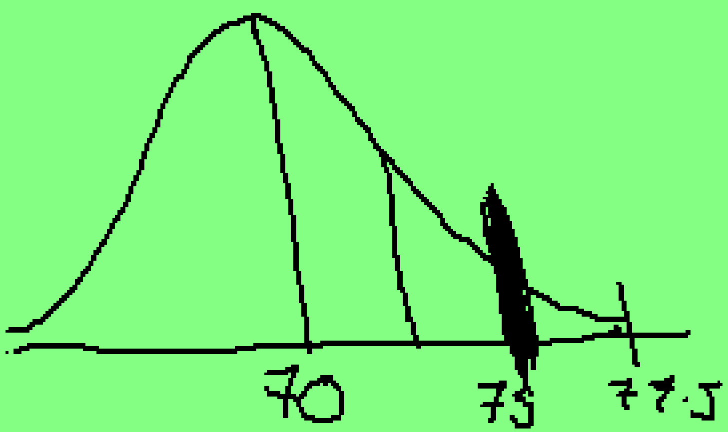
History: 81% > 5%   *Math:* 75% > 5%
mean: 76%   *mean:70%*

3) Same scenario. However how your teacher gives you the class standard deviations. Which class did you do better in?



History:
std. dev: 8%

Math:
std. dev:2.5%



Standardizing Observations

Question: _How can we compare one distribution to another distribution if they don't have the same parameters (mean and std. dev)?_

Answer: compare the obs. to its own μ and σ

To standardize:

- _measure observations...._ in terms of how many σ they are above/below their μ
- Z = z-score

$$Z = \frac{x-\mu}{\sigma} = 2\sigma$$

Hist:

$$Z = \frac{81-76}{8} = 0.625\sigma$$

- _Z- score tells us..._ how many σ an observation is above/below its μ.

$Z = -1.3\sigma$ $\quad Z = 1.3\sigma$

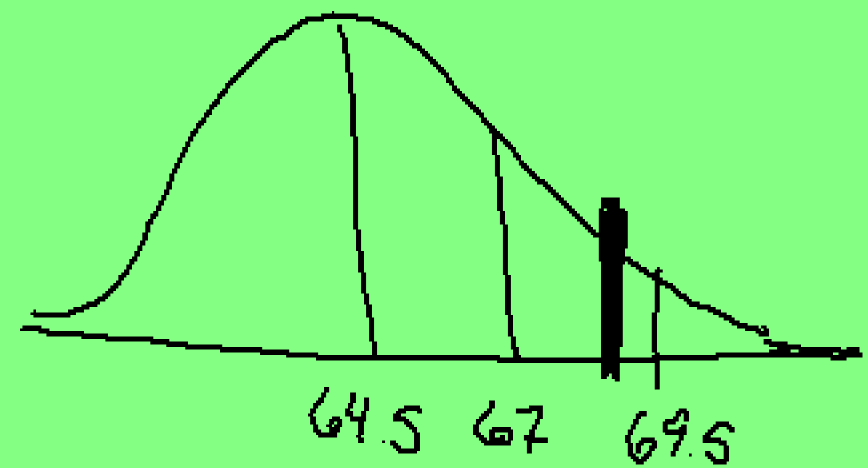
Example: *The heights of 18-24 year old women are normally distributed with the following:*

mean = 64.5" and std. dev = 2.5"

$N(64.5, 2.5)$

We know a woman who is 69" tall. How does she compare to the rest of the women in her age group? Calculate her z score, and interpret what it means.

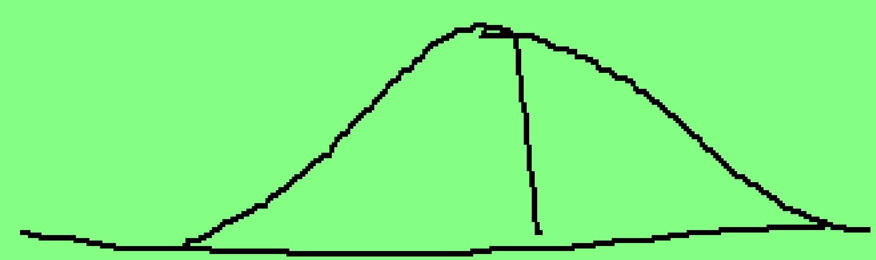
$$z = \frac{x-\mu}{\sigma} = \frac{69-64.5}{2.5} = 1.8\sigma$$



Men: 72"

$N(67, 2.1)$

$$z = \frac{72-67}{2.1} = 2.38\sigma$$



Notes on Standardizing a distribution:

- *Standardizing one observation…* comparing to its mean
- *But we can also standardize…* whole distribution

- *But what does this do to the shape of the distribution?* Same

$$z = \frac{70-70}{2.5} =$$

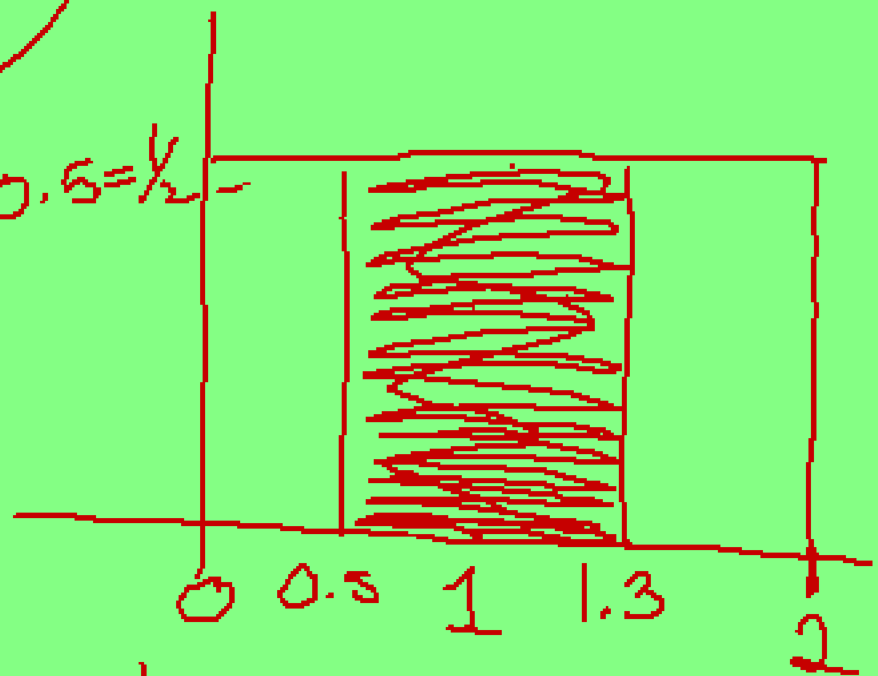
- *Standardizing is actually just…* manipulating

  $-\mu$ $\times \frac{1}{\sigma}$

- *When we do this we have a new distribution, called:* Standard Normal Distrib.

70

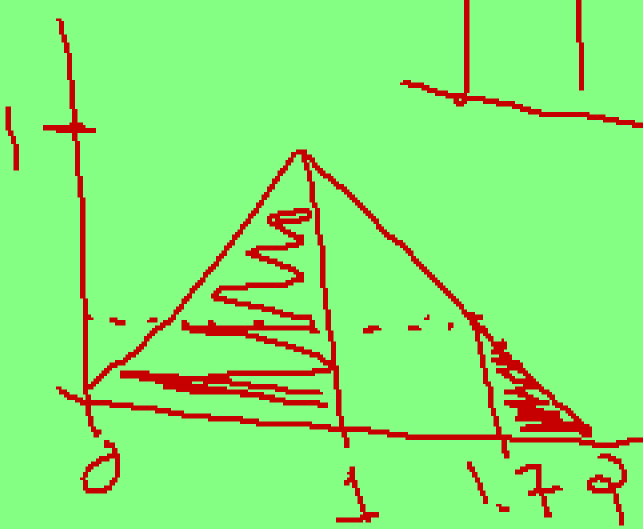
* pop

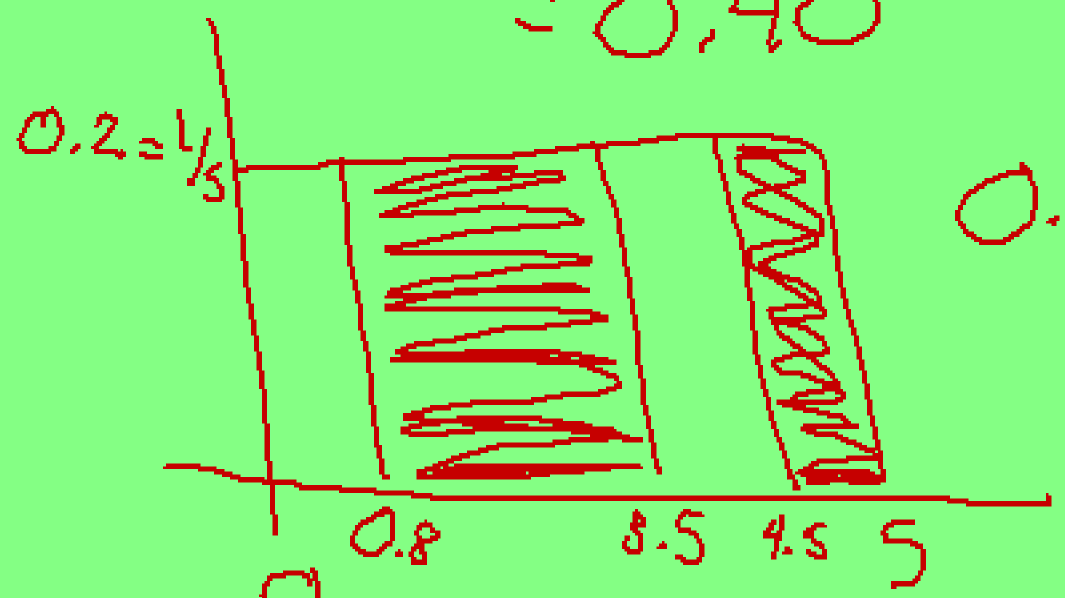


Area = 1 = 100%

$A = (0,8)(0,5)$
$= 0,40$

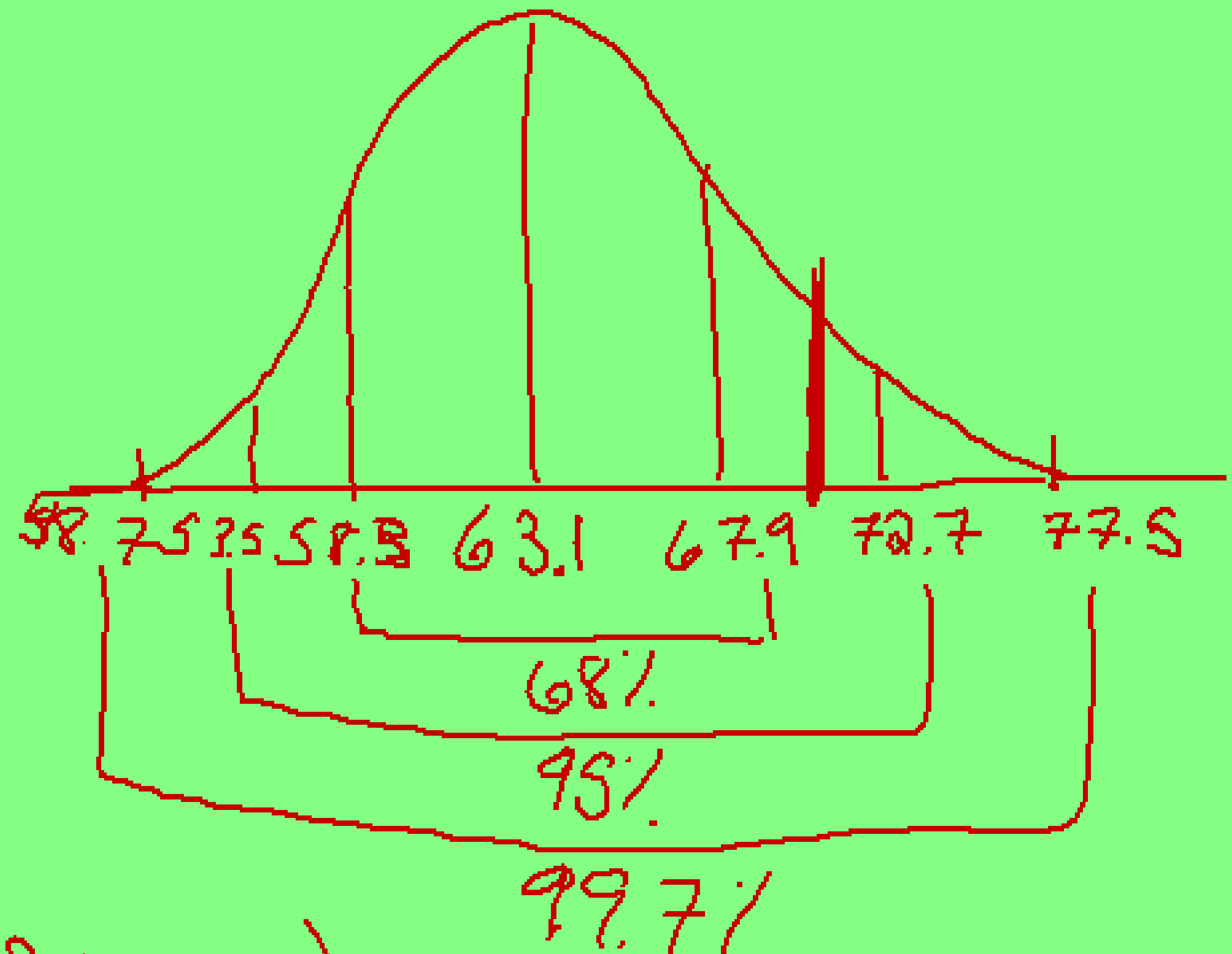
Sample





0.54

74 $N(63.1, 4.8)$



$(58.3, 67.9)$

$P(X < 72.7) = 97.5\%$

$P(X > 67.9) = 16\%$

$P(X < 70) = ?$

- *Table A in the book*
  - *Gives:* % of data (area) below z-score
  - *So, the area to the left of the z-score represents…* % below

$$P(X>70) = 1-0.9251 = 7.49\%$$

$$P(X<70) = 0.9251 = 92.51\%$$

$$z = \frac{70-63.1}{4.8} = 1.4375$$

$$1.44$$

Back to the height example….

Remember that the heights of 18-24 year old women are N(64.5", 2.5"). What percentile is the girl who is 68" tall?

$P(X < 68") = P(z < 1.4) = 0.9192 = \boxed{91.92 \text{ percentile}}$

percentile → % below

$z = \frac{68 - 64.5}{2.5} = 1.4$

What percent of 18-24 year old women are less than 5 feet tall?

$P(X < 60") = P(z < -1.8) = \boxed{0.0359 = 3.59\%}$

$z = \frac{60 - 64.5}{2.5} = -1.8$

What percent 18-24 year old of women are over 5'8" tall?

$P(X > 68") = P(z > 1.4) = 1 - 0.9192 = \boxed{0.0808 = 8.08\%}$

***** PROBABILITY NOTATION!!*** $P(X \geq \_\_\_) = P(Z \leq\geq \_\_\_)$

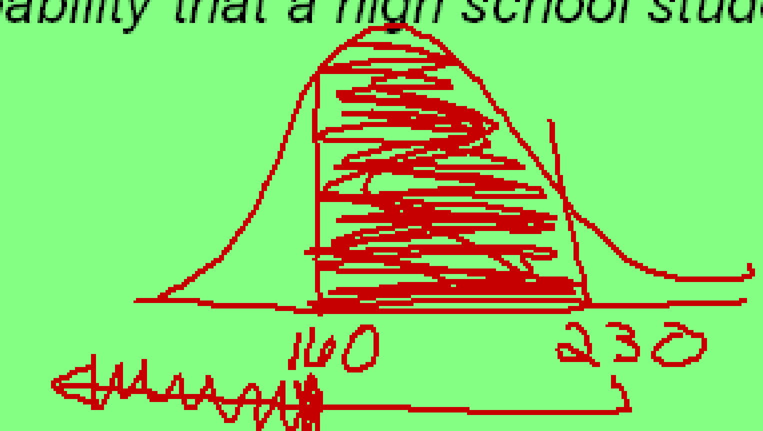
Another example:

Blood pressures of high school students are N(170, 30). What is the probability that a randomly selected high school student has a blood pressure of 180 or above?

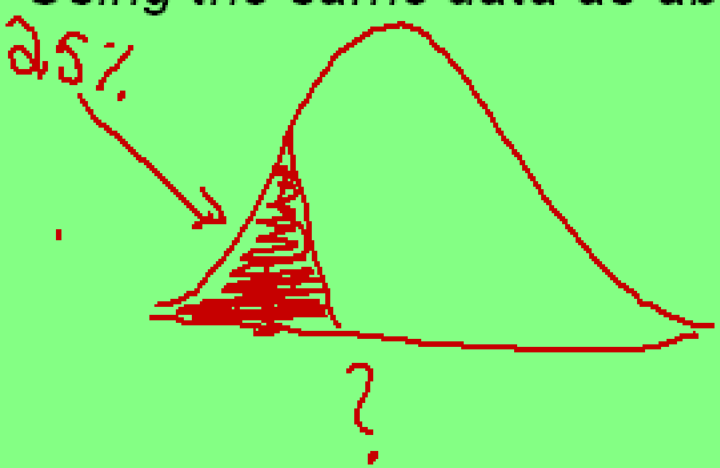
$P(X \geq 180) =$

Using the same data as above, what is the probability that a high school student has a blood pressure between 160 and 230?

$P(160 < X < 230)$



Using the same data as above, what blood pressure has 25% of the observations below it?



$P(X < ?) = 0.25$

$? = 149.765$

30% above?

Calculator use:

To find the percent of observations between 2 points:

$P(160 < X < 230) = 0.608$

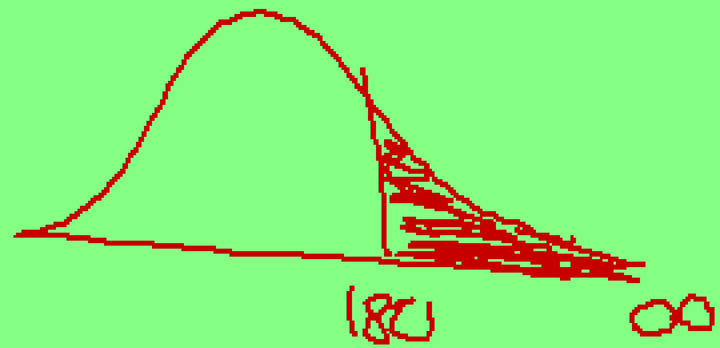
normalcdf(lower bound, upper bound, μ, σ)

$P(x > 180)$

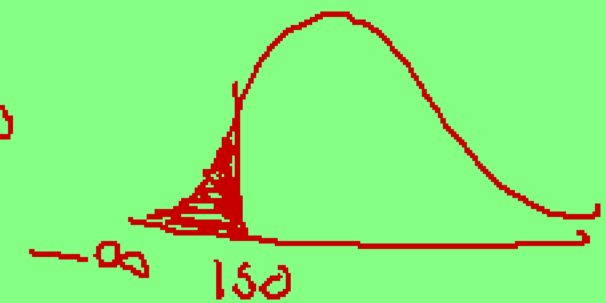
To find what observation has a certain percent of the data below it:

invnorm(% below, μ, σ)

decimal



$P(x < 150)$



0.252

On the calculator, infinity is:

E99

−E99

3 d